Columbia Business School

B9119 – Foundations of Stochastic modeling
Course Logistics - Spring 2023

Teaching Staff

Professor: Assaf Zeevi
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Email: Office location / hours: See Canvas

TA:
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Email: Office location / hours: See Canvas

Venue

- Lectures:
  - When:
  - Where:
- Review Sessions:
  - When: Mostly Fridays 12:00-2:00 (see Canvas)
  - Where: Kravis Hall, for room details see Canvas (or Zoom details if applicable)

Course web page

- navigate via the GSB course portal https://canvas.gsb.columbia.edu/

Columbia Business School

B9119: Syllabus

Foundations of Stochastic Modeling — Professor Assaf Zeevi

General information

This course covers basic concepts and methods in applied probability and stochastic modeling. A unifying theme in the course will be the use of *asymptotic methods* which constitute a powerful tool for the study of complex stochastic systems. Topics covered include: limit theorems (almost sure convergence, weak convergence, and concentration of measure) both in the "standard" iid setting as well as martingales and ergodic sequences; stability of stochastic systems and queueing models; regenerative and renewal limit theorems; and stochastic process limits. The intended audience is master's and doctoral students in programs such as EE, CS, IEOR, Statistics, Mathematics, and those in the DRO division in the Business School. In terms of prerequisites, basic familiarity with probability theory and stochastic processes will be assumed. The exposition will be (mostly) rigorous, yet intentionally skirting some measure-theoretic details; for those interested in such details they can be found in measure theoretic textbooks and other courses (e.g., Probability Theory I/II given in the statistics/math department).

Text and Grading

There is no official textbook. Material is covered in the lectures, with some pointers made to the following reference texts.

- *Probability With Martingales*, D. Williams, Cambridge university press, 1997.
  [Covers probability with, as the name suggest, emphasis on martingales.]
- *Probability: Theory and Examples*, R. Durrett, Duxbury press, 1996.
  [Covers probability with, as the name suggest, many examples.]
- *A First Course in Stochastic Processes*, S. Karlin and H.M. Taylor, Academic press, 1975.
  [Covers Markov chains, martingales and Brownian motion.]
- *Applied Probability and Queues*, S. Asmussen, Springer 2003.
  [Covers applied probability models with emphasis on queueing related processes.]

There will be 6 hand-in assignments over the course of the semester, with spacing of a week or two weeks. These assignments may also contain a simulation-based exercise that will require some simple programming (can be done using MATLAB / Python).
Important note: 1) Students are encouraged to cooperate on the homework assignments, but each assignment should be handed in individually, and any such collaboration should be clearly noted.

2) The only reference material allowed for consultation on these homeworks are textbooks and notes from Columbia courses. Whenever such results are used, they must be clearly referenced!!

There will be a 24 hour take-home final exam at the end of the course at a date that will negotiated later.

The final grade is determined 50% by Homework, and 50% by the final exam mark.

Course overview and main topics covered

The course will cover the following key concepts:

- **Probability:** Essential tools in probability and convergence concepts. Borel-Cantelli lemmas; almost sure convergence; convergence in probability and in expectation; strong law of large numbers (SLLN); interchange arguments; an overview of weak convergence; the central limit theorem (CLT). Introduction to large deviations and refinement
- **Random walks and Martingales:** Stopping times; Wald identities; conditioning and information; basic concepts in martingales; definitions and examples; limit theory; Doob's optional stopping theorems. Limit theory for martingales (MG convergence theorem, SLLN and CLT). Concentration of measure.
- **Stability analysis and intro to queueing models:** Stability of Markov chains via Lyapunov methods. Sample path methods: rate-stability and "long run" behavior; generalized Little's law; coupling and convergence in total variation. Introduction to Markovian queueing models.
- **Renewal theory and regenerative processes:** renewal theory and regeneration. Baby and pointwise renewal theorems; central limit refinements. The key renewal theorem. Regenerative processes; regenerative ratio formula; limit theory and approximations.
- **Functional limit theorems:** Functional strong law. Donsker's theorem. Random walk and Brownian motion; strong approximations; diffusion limits. Heavy traffic scaling.

Outline of lectures

Lecture 1

Probability: preliminaries; Borel-Cantelli lemmas; convergence concepts. Interchange arguments.

Lecture 2

Kolmogorov three series theorem. The strong law of large numbers. Weak convergence: basic concepts.

Lecture 3

Weak convergence: characteristic functions; the Central limit theorem. Skorohod representation. Introduction to concentration bounds.

† **HW1 due in class**

Lecture 4

Large deviations and applications. Finite horizon behavior of Random Walks (RW); stopping times; stopped RW's.

Lecture 5

Martingales I: Definitions, identities and examples; Stopping times and Doob's optional stopping theorems.

† **HW2 due in class**

Lecture 6

Martingales II: Martingale limit theory; MG convergence theorem; SLLN and CLT; concentration of measure.

† **HW3 due in class**

Lecture 7

Stability theory via Lyapunov functions. The Dynkin Martingale. The sample path approach. Stability analysis and large time behavior: basic methods, coupling, instability. Generalized Little's law and conservation laws.

† **HW4 due in class**.

Lecture 8

Convexity and the nature of queueing delays. Performance analysis. Markovian models and the $M/G/1$ queue. P-K formula

Lecture 9

Renewal and regenerative processes (part 1): Limit theory: "Baby" and pointwise renewal theorems; central limit refinements. The renewal equation: formulation and key renewal theorem (KRT).

† **HW5 due in class**.

Lecture 10

Renewal and regenerative processes (part 2): regenerative process structure; the regenerative ratio theorem; limit theory and approximations.

Lecture 11

Stochastic process limits. The functional strong law and the functional central limit theorem (Donsker's theorem). Brownian motion. Heavy-traffic analysis and scaling.

† **HW6 due in class**.

24 hour take-home final scheduled in late April or early May (exact date TBD)